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Eigenstructure Assignment for Fault Tolerant Flight Control Design

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### Abstract

In recent years, fault tolerant flight control systems have gained an increased interest for high performance military aircraft as well as civil aircraft. Fault tolerant control systems can be described as either active or passive. An active fault tolerant control system has to either reconfigure or adapt the controller in response to a failure. One approach is to reconfigure the controller based upon detection and identification of the failure. Another approach is to use direct adaptive control to adjust the controller without explicitly identifying the failure. In contrast, a passive fault tolerant control system uses a fixed controller which achieves acceptable performance for a presumed set of failures.

We have obtained a passive fault tolerant flight control law for the F/A-18 aircraft which achieves acceptable handling qualities for a class of control surface failures. The class of failures includes the symmetric failure of any one control surface being stuck at its trim value. A comparison was made of an eigenstructure assignment gain designed for the unfailed aircraft with a fault tolerant multiobjective optimization gain. We have shown that time responses for the unfailed aircraft using the eigenstructure assignment gain and the fault tolerant gain are identical. Furthermore, the fault tolerant gain achieves MIL-F-8785C specifications for all failure conditions.

# 1 Introduction

## 1.1 Background

Aircraft flight control systems are designed with extensive redundancy to ensure a low probability of failure. During recent years, however, several aircraft have experienced major control system failures. These have caused an increased interest in fault tolerant flight control systems. The objective of fault tolerant flight control is to control and safely land the aircraft in case of severely damaged or inoperable control surfaces. The two approaches to fault tolerant control are active and passive control. An active fault tolerant control system has to either reconfigure or adapt the controller in response to the failure. One way is to reconfigure the controller based upon detection and identification of the failure. Another way is to use direct adaptive control to adjust the controller without explicitly identifying the failure. In contrast, a passive fault tolerant control system uses a fixed controller which achieves acceptable handling qualities for a given set of failures.

Several authors have utilized eigenstructure assignment to design reconfigurable flight control systems. Gavito and Collins [1] used eigenstructure assignment to recover the undamaged modal response under the assumption that the failure has been detected and identified. Napolitano and Swaim [2] used eigenstructure assignment to remove the lateral-longitudinal coupling induced by an asymmetric control surface failure. Jiang [3] used eigenstructure assignment to recover the dominant eigenvalues and eigenvectors of an aircraft longitudinal control system which has undergone some operating condition variations or system component failures. However, all of these approaches require an on-line identification of the parameters of the plant after failure.

Jiang and Zhao [4] used eigenstructure assignment to design passive fault tolerant controllers for actuator failures. This approach is based on robust regional eigenvalue assignment with the addition of a precompensator. However, this design method requires extensive redundancy. Gorrec et. al. [5] proposed a multimodel approach for the landing phase of a large transport aircraft. This method uses a bank of models covering the entire flight envelope. First, the authors design an initial gain for a chosen original model. This gain is used to control all of the models and a multimodel analysis is used to detect the worst model. Then, a quadratic optimization procedure is used to improve the behavior of the worst model while keeping good performance relative to the original model. However, the designer must be careful to avoid conflicting objectives.

We have obtained a new result for eigenstructure invariance when it is known which surface is most likely to fail. We seek a constant gain output feedback controller such that the dominant eigenstructure is invariant under this failure. We show mathemat-

ically that the solution is that the failed surface should not be used. That is, if the  $j^{th}$  control surface will fail, then the  $j^{th}$  row of the constant output feedback gain matrix will be simply zero. For this purpose, first we derive a basis for the subspace in which the eigenvectors of the failed system must lie. Then, we show that there exists a constant output feedback gain matrix which yields an invariant dominant eigenstructure both before and after failure. Finally, we prove that the  $j^{th}$  row of this feedback gain matrix will be zero.

We have obtained a passive fault tolerant flight control for a control surface failure when it is not known in advance which surface will fail. We consider the linearized lateral dynamics of F/A-18 aircraft. The class of control surface failures we consider includes the symmetric failure of any one control surface being stuck at its trim value. It is not known in advance which surface will fail. We computed an optimal feedback gain using an off-line multiobjective optimization technique. A comparison was made of an eigenstructure assignment gain designed for the unfailed aircraft with the fault tolerant multiobjective optimization gain. We have shown that the time responses for the unfailed aircraft using the eigenstructure assignment gain and the fault tolerant gain are identical. Furthermore, the fault tolerant gain achieves MIL-F-8785C specifications for all failure conditions.

## 1.2 Contributions of the Work

- We have a new result for eigenstructure invariance when it is known which surface is most likely to fail. We derived a basis for the subspace in which the eigenvectors of the failed system must lie. We showed mathematically that, if the  $j^{th}$  control surface will fail, then the  $j^{th}$  row of the constant output feedback gain matrix will be simply zero.
- We have obtained a passive fault tolerant flight control for a control surface failure when it is not known in advance which surface will fail. We used an off-line multi-objective optimization technique to design an optimal control for a predefined class of control surface failures. We have shown that the time responses for the unfailed aircraft using the eigenstructure assignment gain and the fault tolerant gain are identical. Furthermore, the fault tolerant gain achieves MIL-F-8785C specifications for all failure conditions.

## 2 Application of Eigenstructure Assignment to Flight Control Design

### 2.1 Literature Review

Consider an aircraft modelled by the linear time invariant matrix differential equation described by

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where  $x$  is the state vector ( $n \times 1$ ),  $u$  the control vector ( $m \times 1$ ) and  $y$  the output vector ( $r \times 1$ ). It is assumed that the  $m$  inputs and the  $r$  outputs are independent. If there are no pilot commands, the feedback control vector  $u$  equals a matrix times the output vector  $y$ :

$$u = -Fy \tag{3}$$

#### **Theorem [6]:**

Given the controllable and observable system described by Eqs. 1 and 2,  $\max(m, r)$  closed loop eigenvalues can be assigned and  $\max(m, r)$  eigenvectors (or reciprocal vectors by duality) can be assigned with  $\min(m, r)$  entries in each vector arbitrarily chosen using constant output feedback.

Andry et. al. [7] concluded that the eigenvectors  $v_i$  must be in the subspace spanned by the columns of  $(\lambda_i I - A)^{-1}B$ . This subspace is of dimension  $m$  which is equal to the number of independent control variables. Thus, if we choose an eigenvector  $v_i$

which lies precisely in the subspace spanned by the columns of  $(\lambda_i I - A)^{-1} B$ , it will be achieved exactly. In general, however, a desired eigenvector  $v_i^d$  will not reside in the prescribed subspace and hence cannot be achieved.

Ref [7] gives a way to find the "best possible choice" for an achievable eigenvector. This best possible eigenvector is the projection of  $v_i^d$  onto the subspace spanned by the columns of  $(\lambda_i I - A)^{-1} B$  (in the least square sense). In many practical situations, complete specification of  $v_i$  is neither required nor known, but rather the designer is interested only in certain elements of the eigenvector. Thus, assume that  $v_i$  has the following structure:

$$v_i^d = [v_{i1}, x, x, x, x, v_{ij}, x, x, v_{in}]^T$$

where  $v_{ij}$  are designer specified components and  $x$  is an unspecified component. Define, as shown by Andry et. al. [7], a reordering operation  $\{ \}^{R_i}$  such that

$$\{v_i^d\}^{R_i} = \begin{bmatrix} \ell_i \\ d_i \end{bmatrix} \quad (4)$$

where  $\ell_i$  is a vector of specified components of  $v_i^d$  and  $d_i$  is a vector of unspecified components of  $v_i^d$ . The rows of the matrix  $(\lambda_i I - A)^{-1} B$  are also reordered to conform with the reordered components of  $v_i^d$ . Thus,

$$\{(\lambda_i I - A)^{-1} B\}^{R_i} = \begin{bmatrix} \tilde{L}_i \\ D_i \end{bmatrix} \quad (5)$$

Then, as shown by Andry et. al. [7], the achievable eigenvector  $v_i^a$  is given by

$$v_i^a = (\lambda_i I - A)^{-1} B z_i \quad (6)$$

where  $z_i = \tilde{L}_i^\dagger \ell_i$  and where  $(\cdot)^\dagger$  denotes the appropriate pseudoinverse of  $(\cdot)$ .

## 2.2 Problem Statement

We consider the linearized lateral dynamics of the F/A-18A aircraft. The rigid body states are lateral(side) velocity( $v$ ), yaw rate ( $r$ ), roll rate ( $p$ ), and bank angle ( $\phi$ ). These states are augmented with first order actuator dynamics and a yaw rate washout filter to yield a 9<sup>th</sup> order model. The control surfaces are asymmetric stabilator ( $\delta_{sc}$ ), asymmetric trailing edge flaps ( $\delta_{tec}$ ), ailerons ( $\delta_{ac}$ ), and rudder ( $\delta_{rc}$ ). The inputs are stabilator command ( $\delta_{sc}$ ), trailing edge flaps command ( $\delta_{tec}$ ), ailerons command ( $\delta_{ac}$ ), and rudder command ( $\delta_{rc}$ ). The measurements are sideslip angle ( $\beta$ ), washed out yaw rate ( $r_{wo}$ ), and roll rate ( $p$ ). A control surface failure is modeled by setting the corresponding column in the control derivative matrix to zero. Each failure condition corresponds to one control surface being fixed at its trim value.

The aircraft can be described by

$$\dot{x} = Ax + Bu \quad (7)$$

$$y = Cx \quad (8)$$

where  $\dim[x] = n$ ,  $\dim[u] = m$ ,  $\dim[y] = r$ , and where

$$A = \begin{bmatrix} -30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -30 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -30 & 0 & 0 & 0 & 0 & 0 \\ -6.93 & 0 & -2.915 & 34.909 & -0.245 & -646.9 & 0.0285 & 32.189 & 0 \\ -0.396 & -0.835 & -0.896 & -3.26 & 0.00849 & -0.2460 & 0.112 & 0 & 0 \\ 11.86 & 13.06 & 13.14 & 4.4 & -0.0256 & 0.73 & -2.83 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & -0.5 \end{bmatrix} \quad (9)$$

$$B^T = \begin{bmatrix} 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1/650 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (11)$$

Observe that Eqs. (9)-(11) may be rewritten compactly as

$$A = \begin{bmatrix} -\delta I_m & 0_{m \times (n-m)} \\ A_{21} & A_{22} \end{bmatrix} \quad (12)$$

$$B = \delta \begin{bmatrix} I_m \\ 0_{(n-m) \times m} \end{bmatrix} \quad (13)$$

$$C = \begin{bmatrix} 0_{r \times m} & (C_{12})_{r \times (n-m)} \end{bmatrix} \quad (14)$$

and the aircraft with a control surface failure may be described by

$$\dot{x} = A_f x + Bu \quad (15)$$

$$y = Cx \quad (16)$$

where

$$A_f = A + \Delta A \quad (17)$$

$$\Delta A = \begin{bmatrix} 0_m & 0_{m \times (n-m)} \\ -\Delta A_{21} & 0_{(n-m) \times (n-m)} \end{bmatrix} \quad (18)$$

$$\Delta A_{21} = \begin{bmatrix} 0 & \cdots & a_{(m+1)j} & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & \cdots & a_{nj} & \cdots & 0 \end{bmatrix} \quad (19)$$

### 2.3 Eigenstructure Invariance for an Anticipated Flight Control Surface Failure

Consider an LTI plant which is augmented with first order actuator dynamics. Suppose we know which control surface is most likely to fail. We seek a constant gain output feedback controller such that the dominant eigenstructure is invariant under this failure. We will show mathematically that the solution is that the surface which will fail should not be used. That is, if the  $j^{th}$  control surface will fail, then the  $j^{th}$  row of the constant output feedback gain matrix will be zero. We present some interesting intermediate results. First we derive a basis for the subspace in which the eigenvectors of the failed system must lie. Then we show that a constant output feedback gain matrix exists which yields an invariant dominant eigenstructure both before and after failure. Finally we show that the  $j^{th}$  row of this feedback gain matrix will be zero.

**Lemma 1:**

Let  $\lambda_i^d$ ,  $i = 1, 2, \dots, r$  be the set of desired eigenvalues.

A basis for the  $m$ -dimensional subspace spanned by the columns of  $(\lambda_i^d I - A_f)^{-1} B$  is given by

$$\begin{bmatrix} \alpha_1 \delta \\ 0 \\ \vdots \\ 0 \\ \hline \gamma_1 \end{bmatrix}, \dots, \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \alpha_j \delta \\ 0 \\ \vdots \\ 0 \\ \hline \gamma_j \end{bmatrix}, \dots, \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \alpha_m \delta \\ \hline \gamma_m \end{bmatrix} \quad (20)$$

where  $\alpha_k = \frac{(\lambda_i^d + \delta) \delta \det(A_{kk}^1)}{\det(A^1)}$ ;  $k = 1, \dots, m$

and  $\gamma = [\gamma_1, \dots, \gamma_m] = \delta(\text{Adj}(A^1))_{21}$

**Proof of Lemma 1:**

$$(\lambda_i^d I - A_f) = \begin{bmatrix} (\lambda_i^d + \delta) I_m & 0_{m \times (n-m)} \\ -A_{21} + \Delta A_{21} & \lambda_i^d I - A_{22} \end{bmatrix} \quad (21)$$

$$(\lambda_i^d I - A_f)^{-1} = \frac{\text{Adj}(\lambda_i^d I - A_f)}{\det(\lambda_i^d I - A_f)} \quad (22)$$

Let

$$(\lambda_i^d I - A_f) = A^1 \quad (23)$$

Let  $\text{Adj}(A^1) = [c_{pq}]^T$  where the cofactors  $c_{pq}$  are given by

$$c_{pq} = (-1)^{p+q} \cdot \det(A_{pq}^1) \quad (24)$$

Due to the special structure of  $A^1$ , it follows that

$$c_{pq} = \begin{cases} (\lambda_i^d + \delta) \cdot \det(A_{pq}^1) & \text{if } p = q, p, q = 1, 2, \dots, m \\ 0 & \text{if } p \neq q, p, q = 1, 2, \dots, m \end{cases} \quad (25)$$

Therefore, the adjoint of  $A^1$  may be written as

$$\text{Adj}(A^1) = \begin{bmatrix} (\lambda_i^d + \delta) \Gamma & (\text{Adj}(A^1))_{12} \\ (\text{Adj}(A^1))_{21} & (\text{Adj}(A^1))_{22} \end{bmatrix} \quad (26)$$

Where  $\Gamma = \text{diag}\{\det(A_{11}^1), \det(A_{22}^1), \dots, \det(A_{mm}^1)\}$

Finally

$$(\lambda_i^d I - A_f)^{-1} B = \frac{\text{Adj}(A^1) \cdot B}{\det(A^1)} = \frac{1}{\det(A^1)} \cdot \begin{bmatrix} (\lambda_i^d + \delta) \delta \Gamma \\ \delta (\text{Adj}(A^1))_{21} \end{bmatrix} \quad (27)$$

Define

$$\alpha = \frac{(\lambda_i^d + \delta) \delta \Gamma}{\det(A^1)} \quad (28)$$

$$\alpha_k = \frac{(\lambda_i^d + \delta) \delta \det(A_{kk}^1)}{\det(A^1)}; \quad k = 1, \dots, m$$

$$\gamma = [\gamma_1, \dots, \gamma_m] = \delta (\text{Adj}(A^1))_{21} \quad (29)$$

Then, the basis vectors are given by (20).

**Theorem 1(Existence):**

There exists a constant output feedback controller  $u = Fy$  such that:

$$(A_f + BFC)v_i^f = (A + BFC)v_i^f = \lambda_i^d v_i^f; \quad i = 1, 2, \dots, r \quad (30)$$

**Proof of Theorem 1:**

Use eq.(15) to obtain

$$(A_f + BFC)v_i^f = (A + \Delta A + BFC)v_i^f = (A + BFC)v_i^f + \Delta A v_i^f \quad (31)$$

Let the  $j^{\text{th}}$  control surface fail. Then

$$\Delta A v_i^f = \begin{bmatrix} 0_m & 0_{m \times (n-m)} \\ 0 \cdots a_{(m+1)j} \cdots 0 \\ \vdots \cdots \vdots \cdots \vdots \\ 0 \cdots a_{nj} \cdots 0 \end{bmatrix} \begin{bmatrix} v_{1i}^f \\ \vdots \\ v_{ni}^f \end{bmatrix} \quad (32)$$

From lemma 1,  $v_i^f$  may be written as

$$v_i^f = \beta_1 \begin{bmatrix} \alpha_1 \delta \\ 0 \\ \vdots \\ 0 \\ \hline \gamma_1 \end{bmatrix} + \cdots + \beta_j \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \alpha_j \delta \\ 0 \\ \vdots \\ 0 \\ \hline \gamma_j \end{bmatrix} + \cdots + \beta_m \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \alpha_m \delta \\ \hline \gamma_m \end{bmatrix} \quad (33)$$

Observe that all the basis vectors have a zero in row  $j$  with the exception of basis vector  $j$ .

Choose  $\beta_j = 0$ . Then,  $\Delta A v_i^f = 0$  and (30) is proven.

**Theorem 2(Gain Computation):**

The constant feedback gain  $F$  from Theorem 1 has all zeros in row  $j$ .

**Proof of Theorem 2:**

Rewriting the eigenvalue-eigenvector equation we obtain

$$(\lambda_i I - A_f) v_i^f = B F C v_i^f \quad (34)$$

We partition (34) conformally mindful of the special structure of  $A_f, B$  matrices.

$$\begin{bmatrix} (\lambda_i + \delta) I_m & 0_{m \times (n-m)} \\ -A_{21} + \Delta A_{21} & \lambda_i I_{(n-m)} - A_{22} \end{bmatrix} \begin{bmatrix} Z_i \\ W_i \end{bmatrix} = \delta \begin{bmatrix} I_m \\ 0_{(n-m) \times m} \end{bmatrix} F C \begin{bmatrix} Z_i \\ W_i \end{bmatrix} \quad (35)$$

Where

$$v_i^f = \begin{bmatrix} Z_i \\ W_i \end{bmatrix} \quad (36)$$

$$\begin{bmatrix} (\lambda_i + \delta) I_m & 0_{m \times (n-m)} \end{bmatrix} \begin{bmatrix} Z_i \\ W_i \end{bmatrix} = \delta \begin{bmatrix} I_m \\ 0_{m \times (n-m)} \end{bmatrix} F C \begin{bmatrix} Z_i \\ W_i \end{bmatrix} = \delta F C \begin{bmatrix} Z_i \\ W_i \end{bmatrix} \quad (37)$$

$$(\lambda_i + \delta) I_m Z_i = \delta F C v_i^f; i = 1, \dots, r \quad (38)$$

Thus

$$\left(\frac{\lambda_i + \delta}{\delta}\right) Z_i = F C v_i^f; i = 1, \dots, r \quad (39)$$

Let

$$Z = \begin{bmatrix} \left(\frac{\lambda_1 + \delta}{\delta}\right) Z_1, & \left(\frac{\lambda_2 + \delta}{\delta}\right) Z_2, & \dots, & \left(\frac{\lambda_r + \delta}{\delta}\right) Z_r \end{bmatrix} \quad (40)$$

$$v^f = \begin{bmatrix} v_1^f, & v_2^f, & \dots, & v_r^f \end{bmatrix} \quad (41)$$

Therefore

$$Z = F C v^f \quad (42)$$

and

$$F = Z (C v^f)^{-1} \quad (43)$$

Finally,

$$Z_i = \begin{bmatrix} x \\ \vdots \\ x \\ 0 \\ x \\ \vdots \\ x \end{bmatrix} \text{ where the zero is in row } j; j \leq m \quad (44)$$

So

$$F = \begin{bmatrix} & & \vdots & \\ 0 & 0 & \cdots & 0 \\ & & \vdots & \end{bmatrix} (CV_f)^{-1} \quad (45)$$

where the zeros are in row  $j$  of matrix  $Z$ . Then the result follows.

### Example

We use the plant described in Section 2.2. We consider the rudder failure which is modeled by setting the 4<sup>th</sup> column of the control derivative matrix to zero.

First, we design an eigenstructure assignment controller for the unfailed condition by assigning the dutch roll mode to have a damping of 0.707 and a natural frequency of 2.83 r/s. The roll subsidence mode is assigned to its open loop value of -2.76. The desired dutch roll eigenvectors are chosen for a sideslip and yaw rate mode which is decoupled from the roll rate and bank angle. The desired roll subsidence eigenvector is chosen for a roll rate mode which is decoupled from vertical velocity and yaw rate. Table 1 shows the output feedback matrix computed for the unfailed aircraft. The initial condition responses using the eigenstructure assignment gain computed for the no failure condition are shown in Figure 1. Observe the degradation in the response after failure.

Next, we compute an optimal feedback gain using Theorem 2 by imposing simultaneous specifications on the unfailed and rudder failure conditions. We require the dutch roll and roll modes for both the unfailed and failed conditions to be the same as for the eigenstructure assignment controller. Furthermore, we require that the fourth row of the dutch roll and roll eigenvectors be zero for both the unfailed and failed conditions. Table 2 shows the output feedback gain matrix. Observe that the fourth row of this gain matrix is exactly zero. The initial condition responses using the optimal gain are shown in Figure 2. Observe that the eigenstructure invariance is achieved but with increased coupling to roll rate and bank angle. To reduce the coupling, the number of eigenvector objectives is reduced by only requiring the fourth row of the dutch roll eigenvector to be zero. The feedback gain matrix is shown in Table 3 where we observe that the gains in the fourth row are small but nonzero. The initial condition responses using this optimal gain are shown in Figure 3. We observe that the coupling to roll rate and bank angle has been significantly reduced, albeit at the expense of exact eigenstructure invariance.

Table 1: Gain matrix computed for no failure

$\beta$	$r_{wo}$	$p$	
0.2412	0.0562	-0.0009	stabilator
0.1317	0.1742	0.0038	trailing edge flap
0.1352	0.1918	0.0047	aileron
-0.8366	0.9019	0.0379	rudder

Table 2: Gain using theorem (dutch roll and roll modes)

$\beta$	$r_{wo}$	$p$	
-9.1724	-5.6198	-0.2616	stabilator
4.7829	4.8542	0.1423	trailing edge flap
-13.0675	5.2913	0.2041	aileron
0	0	0	rudder

Table 3: Gain using theorem (dutch roll only)

$\beta$	$r_{wo}$	$p$	
8.2678	-7.6184	-0.1597	stabilator
-4.0159	2.7116	0.0564	trailing edge flap
-3.9988	4.5213	0.0944	aileron
0.0613	0.0298	0.0198	rudder

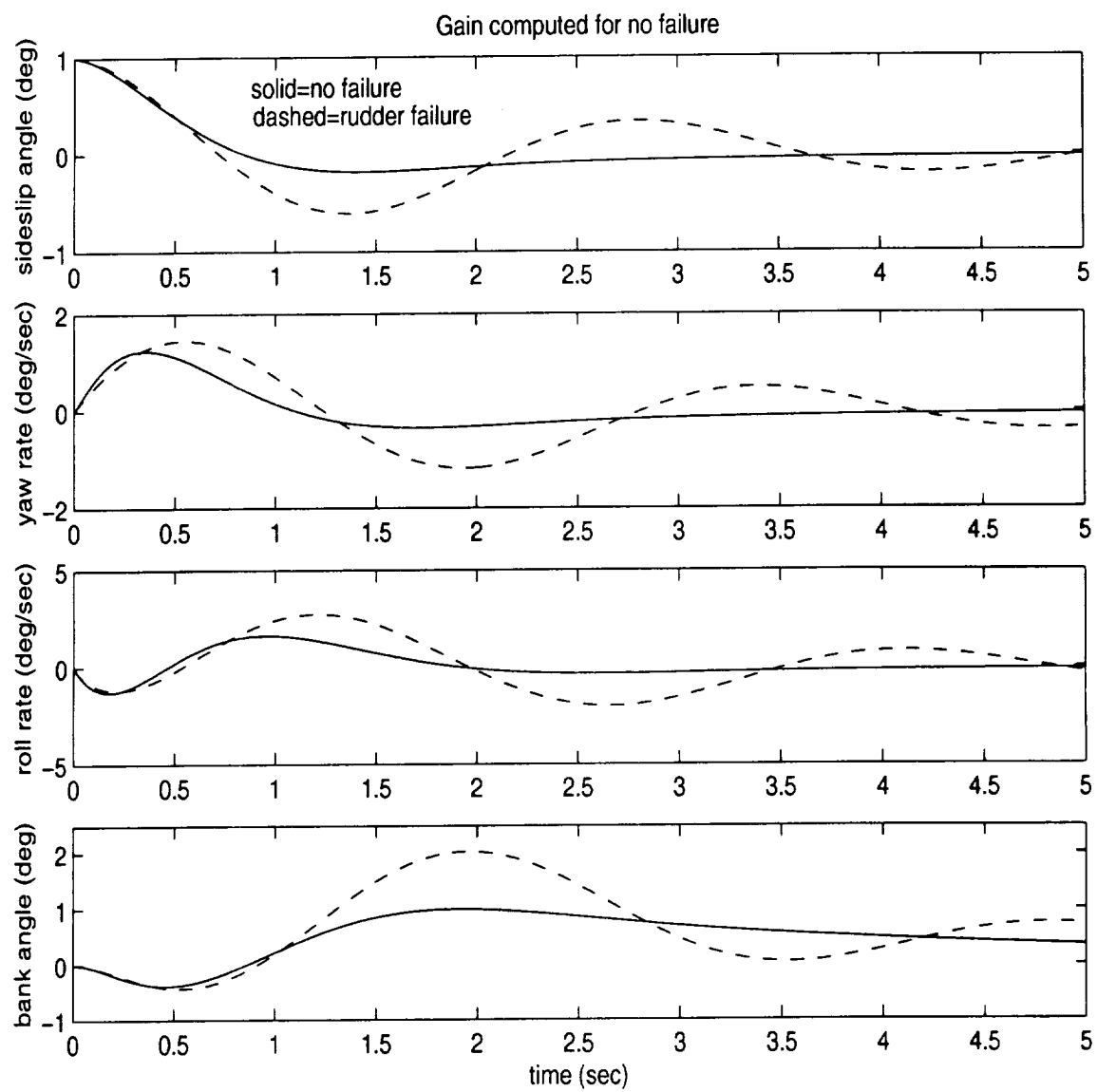


Figure 1: Gain computed for no failure

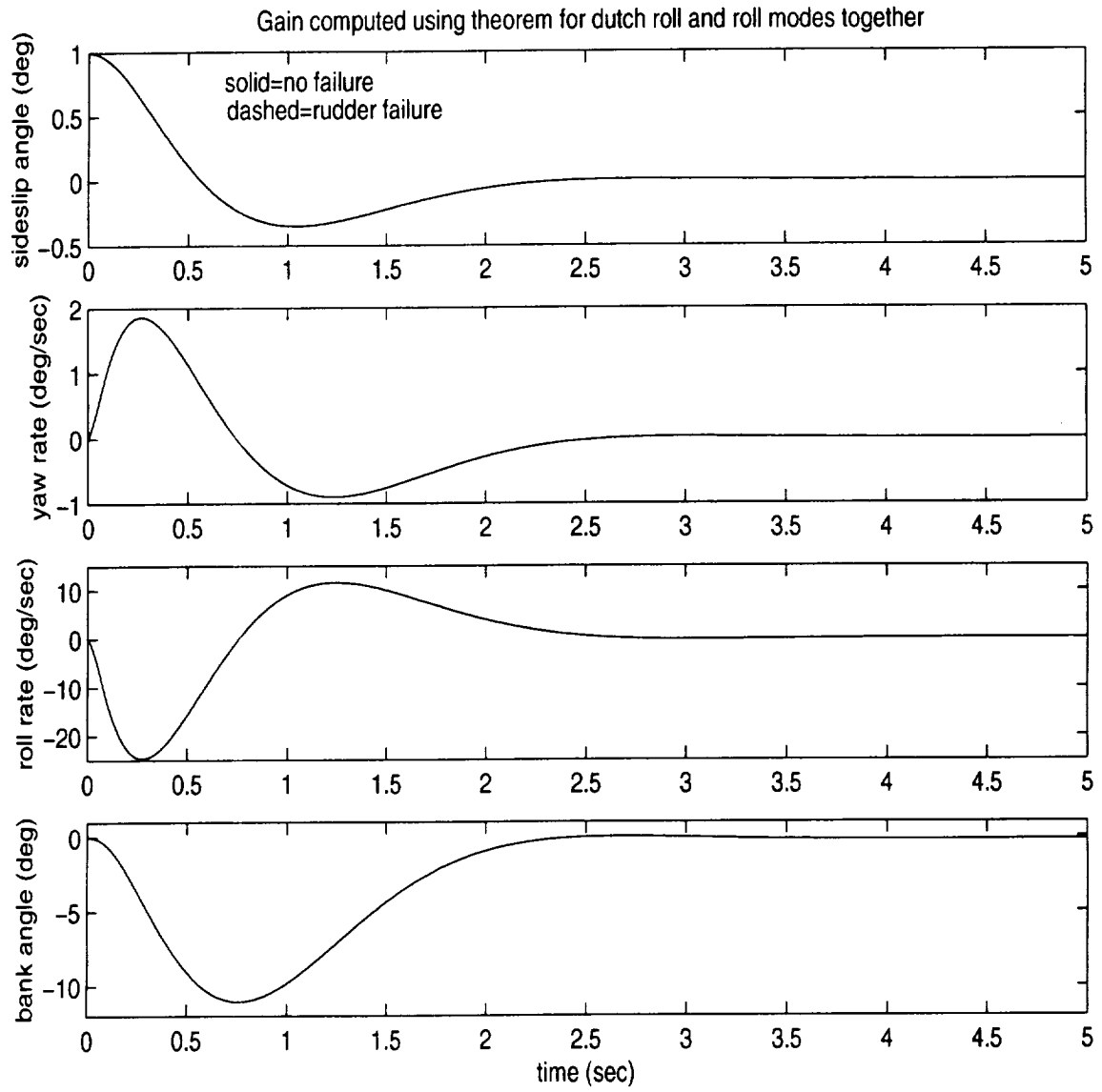


Figure 2: Gain computed using theorem for dutch roll and roll

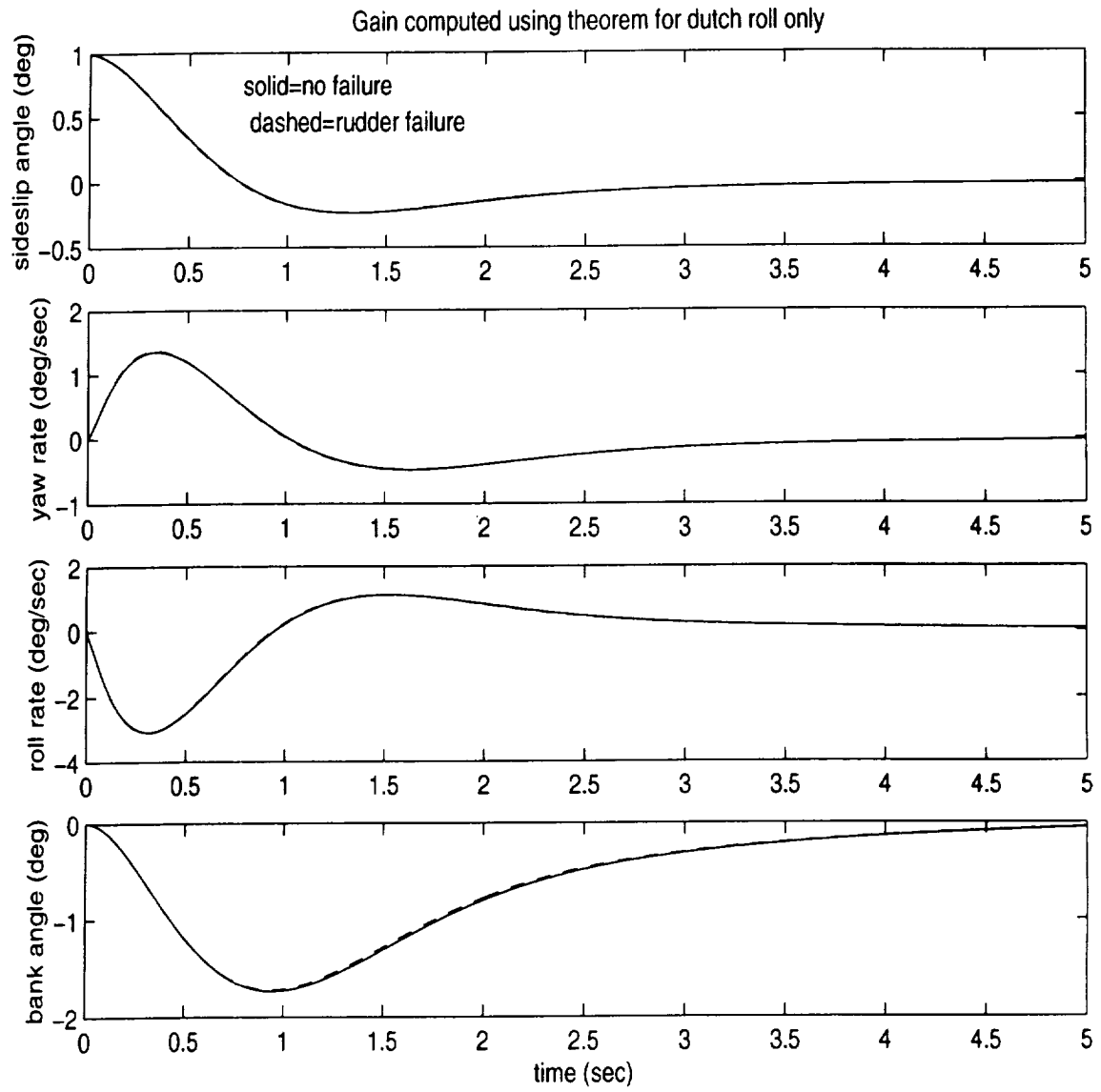


Figure 3: Gain computed using theorem for dutch roll mode

## 2.4 Passive Fault Tolerant Control for an Unknown Control Surface Failure

We have designed an optimal gain which is tolerant to a control surface failure. The class of control surface failures we consider includes the symmetric failure of any one control surface with the surface being stuck at its trim value. It is not known in advance which surface will fail. We compare an eigenstructure assignment gain designed for the unfailed aircraft with a multiobjective optimization gain designed to achieve (1) the same response for the unfailed condition as was obtained using eigenstructure assignment and (2) MIL-F-8785C specifications for all failure conditions belonging to our class.

We use program ATTGOAL from the MATLAB Optimization Toolbox [8] to solve the multiobjective goal attainment problem described by minimize  $\gamma$  such that  $x, \gamma$

$$F(x) - w\gamma \leq \text{goal} \quad \text{where } F(x), w, \text{ and goal are given.}$$

The function  $F(x)$  is the objective function to be minimized at the point  $x$ . ATTGOAL attempts to minimize the function values to attain the goal values given by goal. Alternatively, to make an objective function as near as possible to a goal value, options(15) is set equal to the number of objectives required to be in the neighborhood of goal values. These objectives must be partitioned into the first elements of the function  $F(x)$ . Goal is a vector of values that the objectives attempt to attain. The weighting vector  $w$  is used to control the relative under-attainment or over-attainment of the objectives. The weighting function may be set to  $w = \text{abs}(\text{goal})$  to obtain the same percentage of under or over-attainment. When  $w$  is positive, ATTGOAL attempts to make the objectives less than the goal values. To make the objective functions greater than the goal values,  $w$  is set to be negative.

We consider the linearized lateral dynamics of the F/A-18A aircraft which was described in Section 2.2. We design an eigenstructure assignment controller for the unfailed condition by assigning the dutch roll mode to have a damping of 0.707 and a natural frequency of 2.83 r/s. The roll subsidence mode is assigned to its open loop value of -2.76. The desired dutch roll eigenvectors are chosen for a sideslip and yaw rate mode which is decoupled from the roll rate and bank angle. The desired roll subsidence eigenvector is chosen for a roll rate mode which is decoupled from vertical velocity and yaw rate.

Table 4 shows the dutch roll and roll subsidence mode desired eigenvectors, the eigenvectors from the projection onto the achievable subspaces, and the eigenvectors of  $(A + BFC)$ . First, we remark that the entries in the imaginary part of the dutch roll eigenvectors, which we desire to be zero, are not zero after projecting onto the achiev-

Table 4: Desired and Achievable Eigenvectors (actuator entries not shown)

Desired		Projection		$(A + BFC)$		
D. R.	R.	D. R.	R.	D. R.	R.	
$1 + jx$	0	$1 + j0$	0.0001	$1 - j59.4$	0.0001	$v$
$x + j1$	0	$0.0028 + j1$	-0.0168	$0.0032 + j1$	-0.0168	$r$
$0 + j0$	1	$-0.0137 + j3.83$	1	$-0.012 + j5.24$	1	$p$
$0 + j0$	$x$	$0.0006 + j2.14$	-0.362	$-0.0003 - j2.52$	-0.362	$\phi$
$x + jx$	0	$-0.0008 + j0.034$	0.0037	$-0.0008 + j0.08$	0.0037	$x_g$

able subspace. This will cause coupling to the roll rate response. The reason that we do not achieve the zero eigenvector entries is that the  $\tilde{L}_i$  are poorly conditioned. So to achieve a gain matrix with reasonable magnitude, we discard the smallest singular value when computing the pseudo inverse to obtain  $z_i$ . If less coupling is required then another choice for the desired dutch roll eigenvectors should be made which specifies fewer entries. Second, we remark that we have  $r < m$  which is different from most aerospace applications of eigenstructure assignment which have appeared in the literature. Some authors have suggested assigning the left eigenvectors when  $r < m$ . However, the right eigenvectors still lie in the subspaces spanned by the columns of  $(\lambda_i I - A)^{-1}B$ . Since we only assign  $r$  eigenvalues, we can also assign  $m$  entries in the corresponding eigenvectors. This is independent of the relation between  $r$  and  $m$ .

The feedback gain matrix is shown in Table 5. The sideslip angle responses to a one degree initial sideslip for the unfailed and four failure conditions are shown in Figure 4 where the oscillatory response corresponds to a rudder failure. We remark that the response is unacceptable when a rudder failure occurs. In fact, the dutch roll damping is only  $\zeta = 0.18$  when a rudder failure occurs. This is significantly less than the minimum value of  $\zeta = 0.4$  which is required in MIL-F-8785C. We further remark that the asymmetric stabilator, asymmetric trailing edge flaps, and ailerons are all good producers of rolling moment, but only rudder is a good producer of yawing moment. Thus, a rudder failure is the most difficult condition for achieving desirable handling qualities. Next, we design an optimized controller by imposing

Table 5: Eigenstructure Assignment Feedback Gain Matrix

$\beta$	$r_{wo}$	$p$	
0.2412	0.0562	-0.0009	stabilator
0.1317	0.1742	0.0038	trailing edge flap
0.1352	0.1918	0.0047	aileron
-0.8366	0.9019	0.0379	rudder

simultaneous specifications on the unfailed and four failure conditions. We use the multiobjective optimization method which is implemented in the MATLAB Optimization Toolbox [8] program ATTGOAL. Our objective is for the optimal unfailed responses to be the same as the eigenstructure assignment unfailed responses while also achieving MIL-F-8785C specifications for the failed responses. The MIL-F-8785C specifications for Level 1, Category A (CO and GA), Class IV flight are as follows: dutch roll damping and natural frequency greater than 0.4 and 1.0, respectively; roll subsidence time constant less than 1 second; and spiral time to double amplitude greater than 12 seconds. We remark that the eigenstructure assignment controller achieves all these specifications with the exception of the dutch roll damping which is 0.18 when a rudder failure occurs.

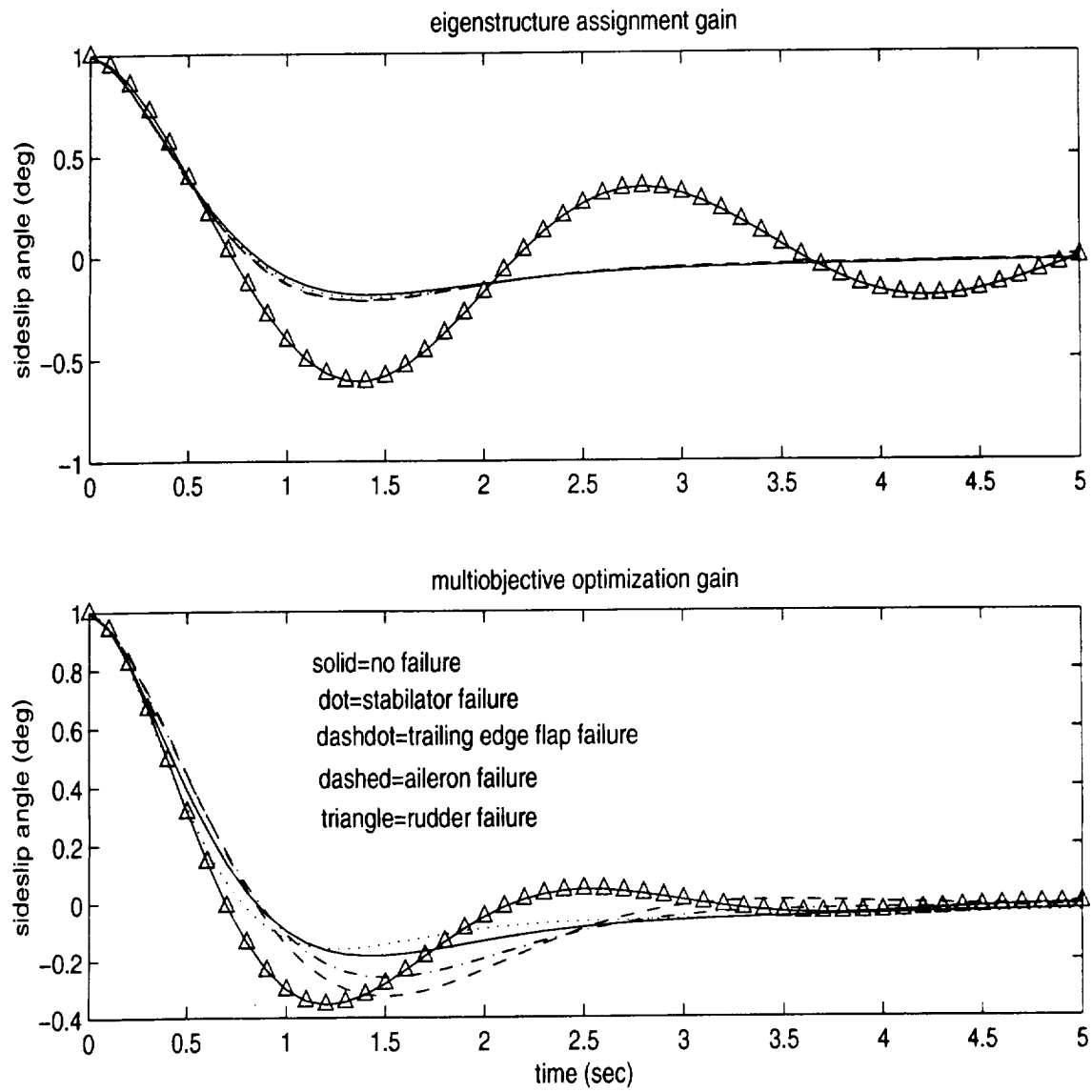


Figure 4: Sideslip angle responses

We compute an optimal feedback gain using the following objectives: (1) the dutch roll and roll subsidence modes are to be the same as for the eigenstructure assignment controller, and (2) the dutch roll and roll subsidence modes are to achieve MIL-F-8785C specifications for the four failure conditions. We remark that we cannot directly specify the spiral mode when using a stability augmentation system (SAS) which does not feedback bank angle.

The optimization parameters are (1) the real part of the dutch roll eigenvalue  $\text{Re}(\lambda_{dr})$ , (2) the imaginary part of the dutch roll eigenvalue  $\text{Im}(\lambda_{dr})$ , (3) the roll subsidence eigenvalue  $\lambda_{roll}$ , (4) the free eigenvector parameters  $z_{roll}$  for the roll subsidence mode, and (5) the free eigenvector parameters  $z_{dr}$  for the dutch roll mode where real arithmetic is used. The parameters are initialized at the values obtained when using the eigenstructure assignment gain.

The objectives and weightings at the unfailed condition are

$$\begin{aligned}\lambda_{roll} &= -2.76 \quad , \quad w = \text{abs}(-2.76) \\ \text{Re}(\lambda_{dr}) &= -2 \quad , \quad w = \text{abs}(-2) \\ \text{Im}(\lambda_{dr}) &= 2 \quad , \quad w = \text{abs}(2)\end{aligned}$$

The objectives and weightings at the 4 failure conditions are

$$\begin{aligned}\zeta_{dr}^i &\geq 0.4 \quad , \quad w^i = -1; \quad i = 1, \dots, 4 \\ (\omega_n)_{dr}^i &\geq 1.0 \quad , \quad w^i = -1; \quad i = 1, \dots, 4\end{aligned}$$

The optimal gain is shown in Table 6 which required 2.04 seconds of CPU time and 52 function evaluations. The sideslip angle response to a one degree initial sideslip is shown in Figure 4. We observe that all responses achieve the desired specifications. The dutch roll damping  $\zeta_{dr}$  is now 0.4 for the rudder failure condition which achieves the MIL-F-8785C specification. The yaw rate responses for an initial sideslip for the

Table 6: Optimal Feedback Gain Matrix

$\beta$	$r_{wo}$	$p$	
5.2056	-2.851	-0.0605	stabilator
-2.1993	1.4108	0.0292	trailing edge flap
-2.651	1.9134	0.0400	aileron
-0.0766	0.4808	0.0293	rudder

eigenstructure assignment and multiobjective optimization gains are shown in Figure

5. Again we observe that the eigenstructure assignment gain exhibits an unacceptable oscillation when rudder failure occurs whereas the multiobjective optimization gain yields an acceptable response. The roll rate responses for the eigenstructure assignment and multiobjective optimization gains are shown in Figure 6. Here we observe that the roll rate response with the multiobjective optimization gain exhibits a significant coupling when a stabilator failure occurs. The bank angle responses for the eigenstructure assignment and multiobjective optimization gains are shown in Figure 7. Again, we observe that the response with the multiobjective optimization gain exhibits a significant coupling when a stabilator failure occurs. However, the MIL-F-8785C specification on sideslip to bank angle coupling is that the minimum  $\zeta\omega_n$  is increased if  $\omega_n^2|\phi/\beta| > 20$ . For our aircraft with a stabilator failure we have  $\omega_n^2|\phi/\beta| < (2.83)^2(2.2) = 17.6 < 20$ . Therefore, our responses meet the MIL specification even though a smaller sideslip to bank angle coupling may be desirable. The responses for the unfailed aircraft to an initial sideslip are shown in Figure 8 for both the eigenstructure assignment and multiobjective optimization gains. We observe that the sideslip and yaw rate responses are identical and there is only an insignificant degradation in the coupling to roll rate. Figure 9 shows the responses using both gains for the unfailed aircraft until time  $t = 0.5$  sec when a rudder failure occurs. We observe that the eigenstructure assignment gain allows oscillation and large settling time whereas the multiobjective optimization gain yields an acceptable response after failure.

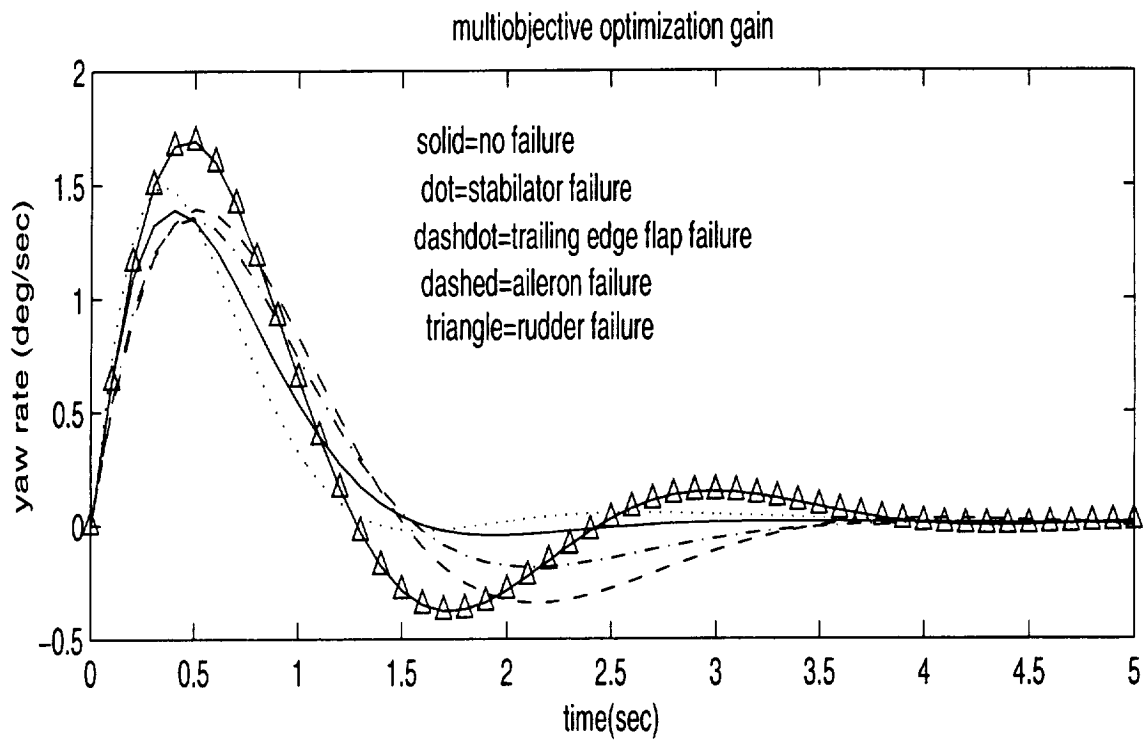
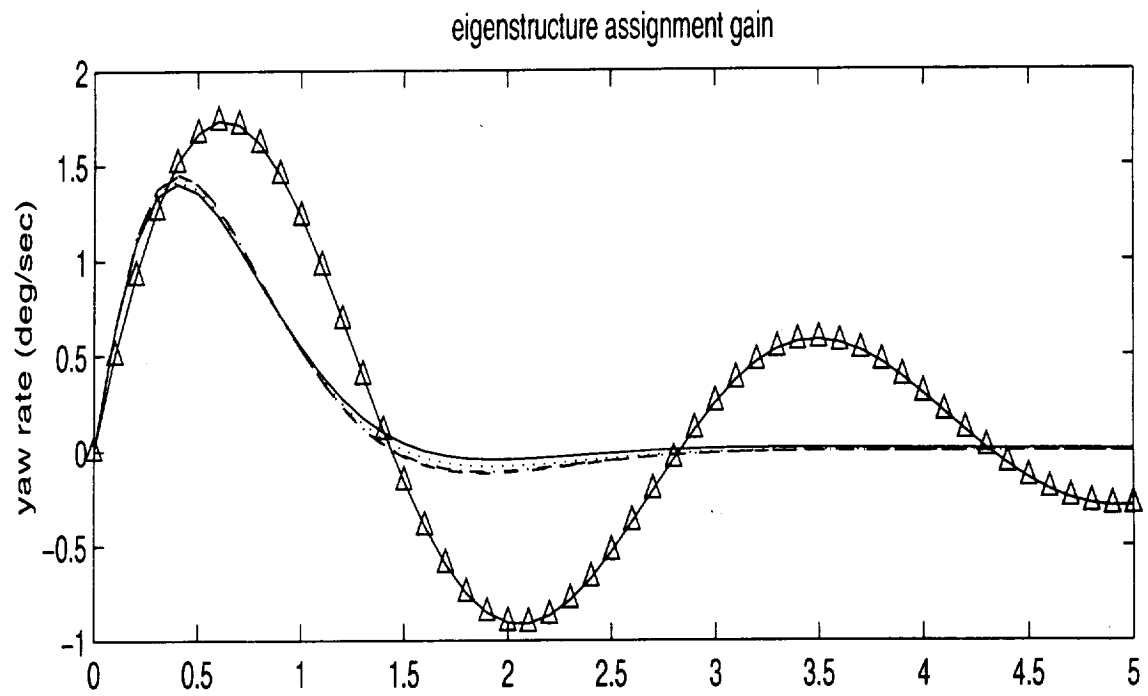


Figure 5: Yaw rate responses

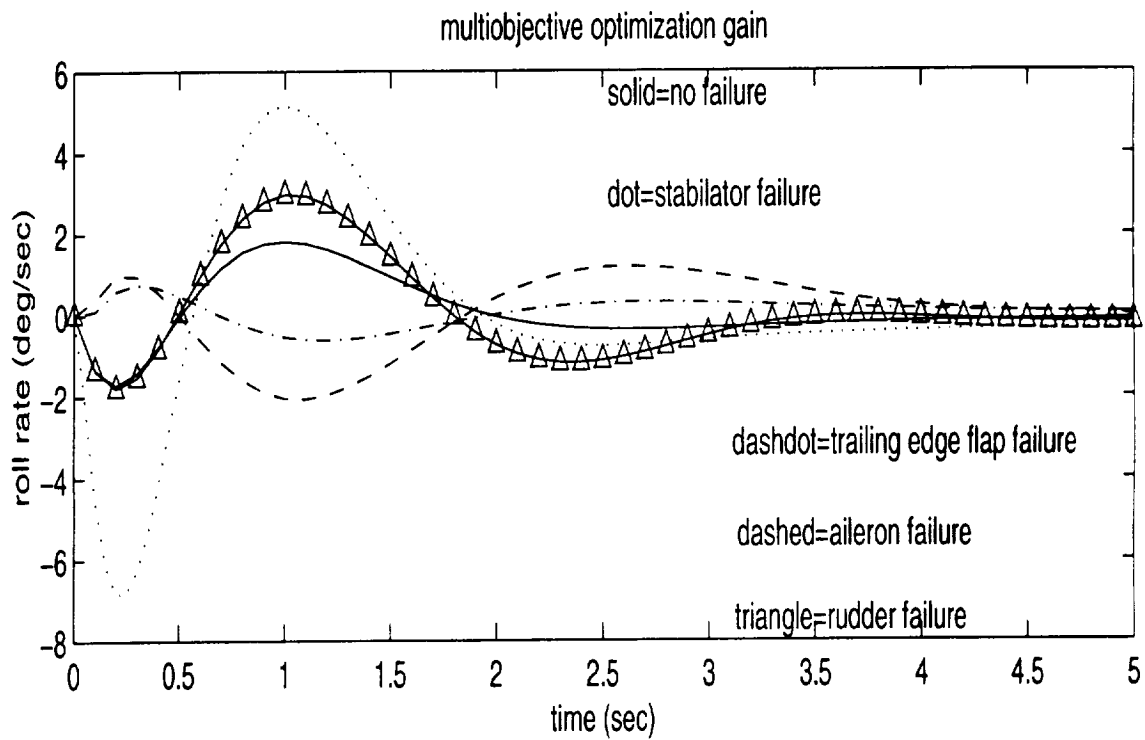
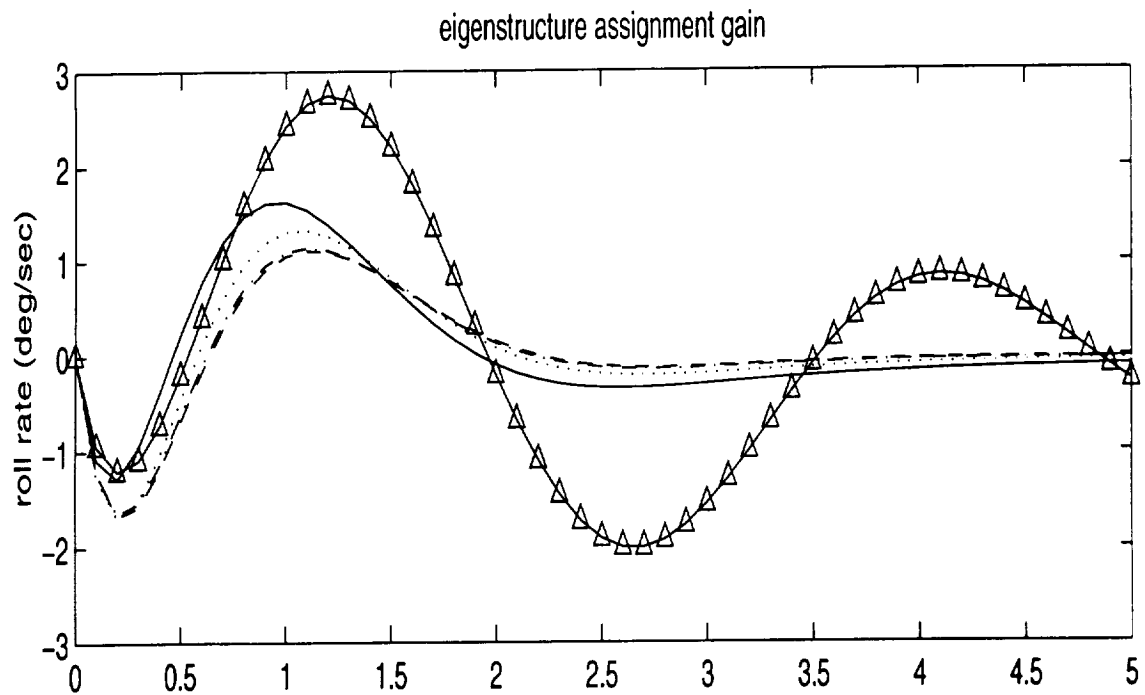


Figure 6: Roll rate responses

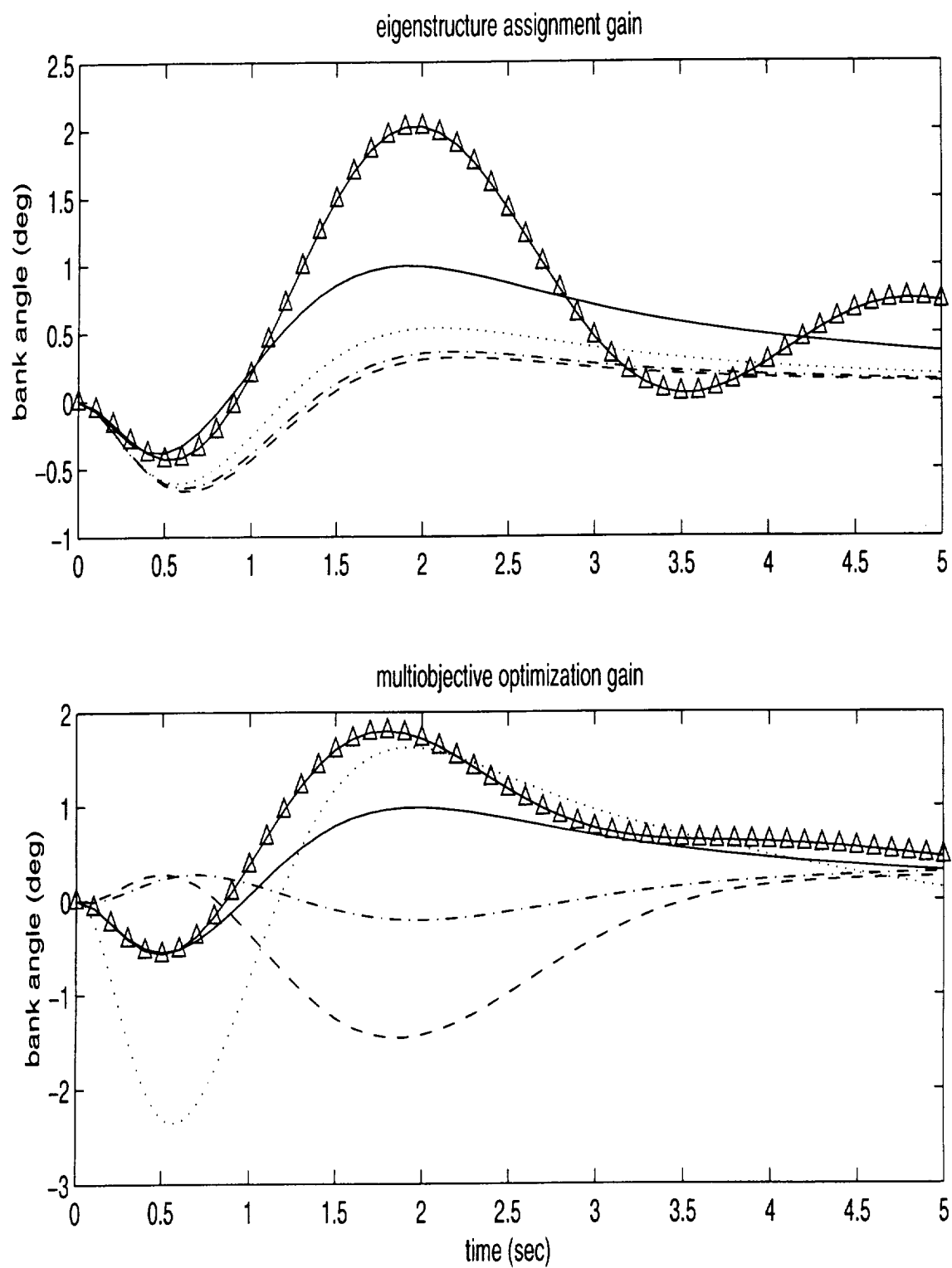


Figure 7: Bank angle responses

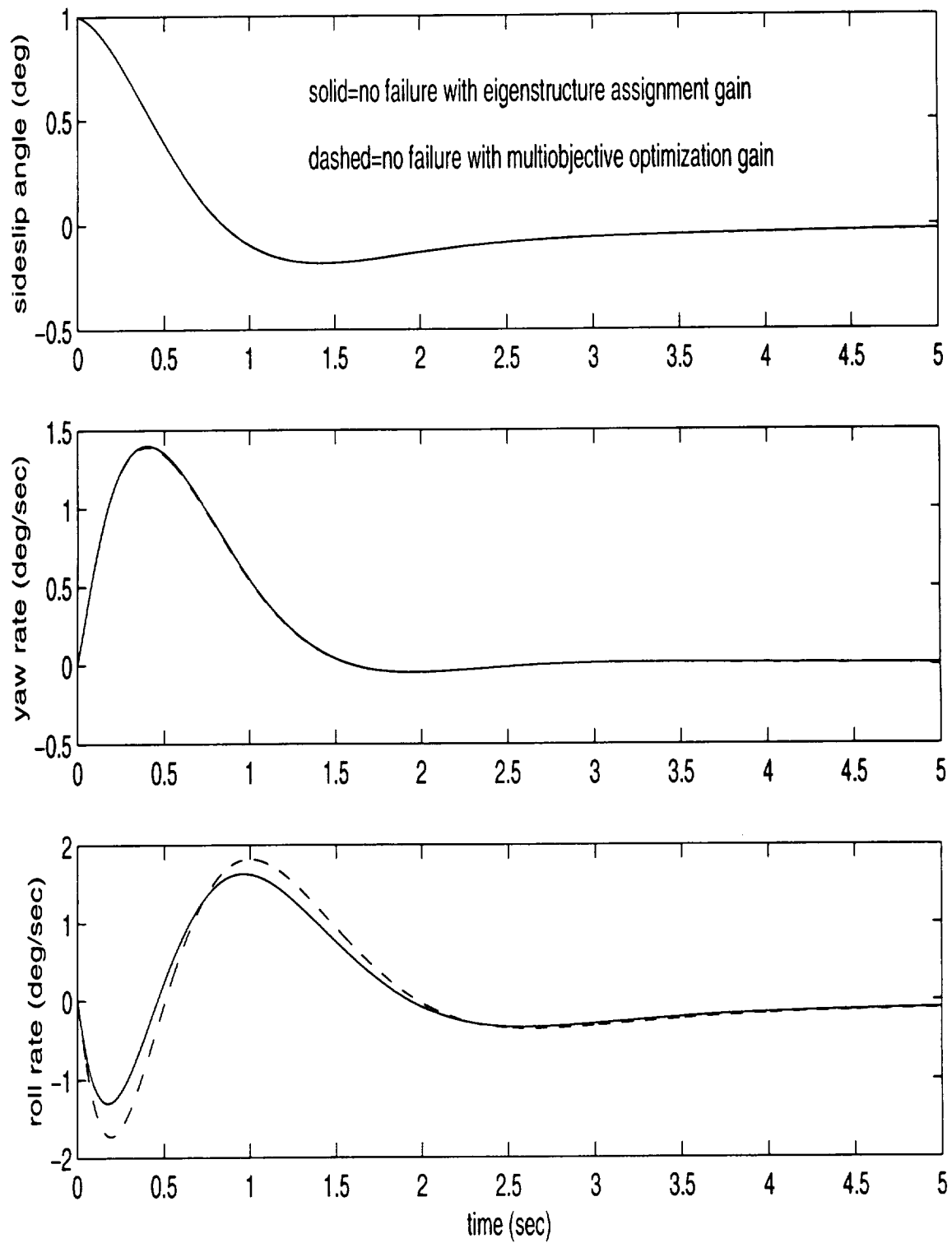


Figure 8: Responses for eigenstructure assignment gain and multiobjective optimization gain with no failure

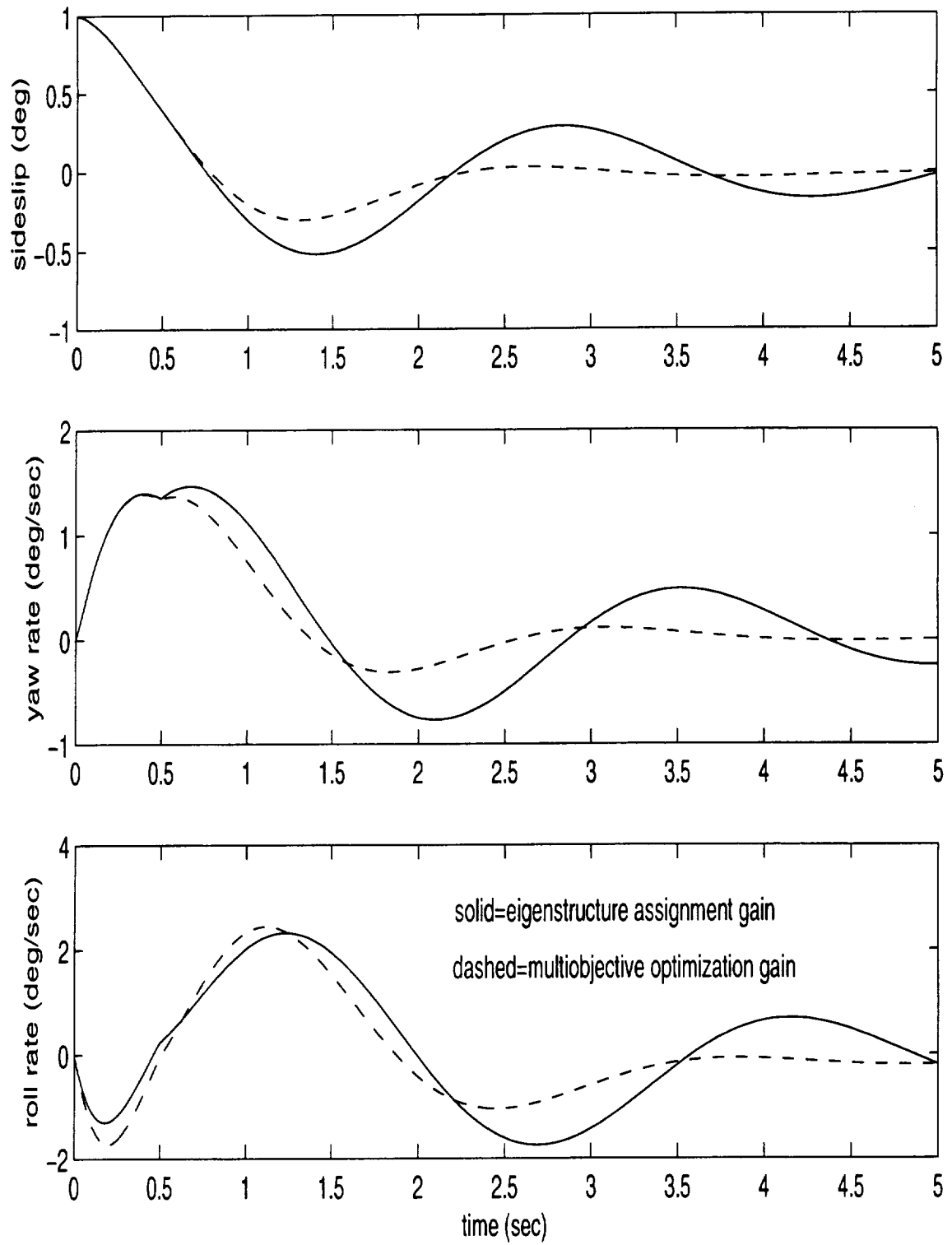


Figure 9: Responses for rudder failure at  $t = 0.5\text{sec}$

### **3 RESEARCH PAPERS**

1. Belkharraz I. Abderrazak and Kenneth M. Sobel, "Fault Tolerant Flight Control for a Class of Control Surface Failure ". Proceedings of the American Control Conference, Chicago, Illinois. June 2000, pp 4209-4213.
2. Belkharraz I. Abderrazak and Kenneth M. Sobel, "Eigenstructure Invariance for an Anticipated Flight Control Surface Failure", Proceedings of the American Control Conference, Arlington, Virginia, June 2001, pp 1833-1835.

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